



Christ Church
Grammar School

2017
UNIT TEST 4

MATHEMATICS SPECIALIST Year 12

Section One: Calculator-free

Student name _____

Teacher name _____

Time and marks available for this section

Reading time before commencing work: 2 minutes
Working time for this section: 15 minutes
Marks available: 15 marks

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,
correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Instructions to candidates

1. Write your answers in this Question/Answer Booklet.
2. Answer all questions.
3. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.
4. It is recommended that **you do not use pencil**, except in diagrams.

Question 1**(6 marks)**

$$\frac{x-5}{x^2-5x+6} = \frac{A}{(x-2)} + \frac{B}{(x-3)}, \text{ where } A \text{ and } B \text{ are real numbers.}$$

(a) Determine the values of A and B .

(3 marks)

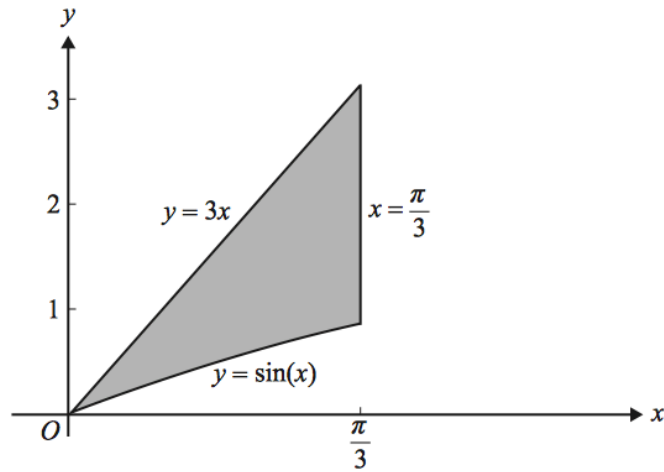
(b) Hence, or otherwise, evaluate $\int_0^1 \frac{x-5}{x^2-5x+6} dx$.

(3 marks)

Question 2

(6 marks)

The shaded region below is enclosed by the graph of $y = \sin(x)$ and the lines $y = 3x$ and $x = \frac{\pi}{3}$.



The region is rotated about the x -axis.
 Determine the volume of the resulting solid of revolution.

Question 3**(3 marks)**

The position vector of a moving particle is given by $\mathbf{r}(t) = \sin\left(\frac{t}{3}\right) \mathbf{i} + \frac{1}{2} \sin\left(\frac{2t}{3}\right) \mathbf{j}, t \geq 0$.

Determine the cartesian equation of the path followed by the particle.



MATHEMATICS SPECIALIST Year 12

Section Two:

Calculator-assumed

Student name _____

Teacher name _____

Time and marks available for this section

Reading time before commencing work: 3 minutes

Working time for this section: 30 minutes

Marks available: 30 marks

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet

Formula Sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, and up to three calculators approved for use in the WACE examinations

Important note to candidates

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Instructions to candidates

1. Write your answers in this Question/Answer Booklet.
2. Answer all questions.
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4. It is recommended that **you do not use pencil**, except in diagrams.

Question 4**(6 marks)**

With \mathbf{i} and \mathbf{j} horizontal and vertical unit vectors respectively, a particle moves with a constant acceleration of $2\mathbf{j} \text{ m/s}^2$.

Let $t, t \geq 0$, be the time in seconds. Initially, i.e. when $t = 0$, the position vector of the body is $(2\mathbf{i} + 5\mathbf{j}) \text{ m}$, and its velocity vector is $(3\mathbf{i} - 7\mathbf{j}) \text{ m/s}$.

Determine the cartesian equation of the path of the particle.

Question 5**(11 marks)**

The position vector $\mathbf{r}(t)$, from the origin O , of a drone t seconds after leaving the ground is given by

$$\mathbf{r}(t) = \left(50 + 25\cos\left(\frac{\pi t}{30}\right)\right) \mathbf{i} + \left(50 + 25\sin\left(\frac{\pi t}{30}\right)\right) \mathbf{j} + \frac{2t}{5} \mathbf{k}$$

where \mathbf{i} is a unit vector to the east, \mathbf{j} is a unit vector to the north and \mathbf{k} is a unit vector vertically up. Displacement components are measured in metres.

- (a) State the time, in seconds, required for the drone to gain an altitude of 60 m. (1 mark)
- (b) After how many seconds will the drone first be directly above the point of take-off? (1 mark)
- (c) Show that the velocity of the drone is perpendicular to its acceleration. (4 marks)

(d) Determine the speed of the drone.

(2 marks)

(e) A treetop has position vector $\mathbf{r}(t) = 60 \mathbf{i} + 40 \mathbf{j} + 8 \mathbf{k}$.

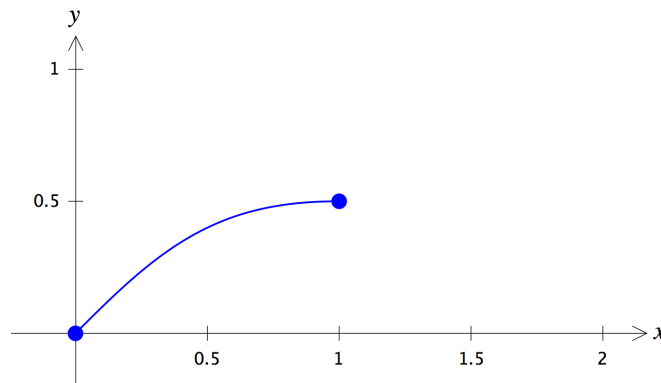
Find the distance of the drone from the treetop after it has been travelling for 45 seconds.

(3 marks)

Question 6

(6 marks)

The diagram shows the graph of $f(x) = \frac{x}{1+x^2}$ for $0 \leq x \leq 1$.



The area bounded by $y = f(x)$, the line $x = 1$ and the x -axis is rotated about the line $x = 1$ to form a solid.

Determine the volume of the solid.

Question 7**(7 marks)**

Let S be the curve in the cartesian plane defined by $\mathbf{r}(t) = \begin{pmatrix} 1+t \\ t^2-3 \end{pmatrix}$, $t \in \mathbb{R}$.

Let T be the curve in the cartesian plane defined by $\mathbf{r}(t) = \begin{pmatrix} 1+2\cos(t) \\ -4+3\sin(t) \end{pmatrix}$, $0 \leq t \leq \pi$.

Calculate the area of the region bounded by the two curves.